

# Placement of Synchronized Measurements in Power Networks for Redundant Observability

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**Abstract:** The rapid expansion of bulk power grids need full monitoring, operation, protection and control. To tackle these challenges, Phasor Measurement Unit (PMU) provides the real time monitoring which improves the reliability of power system and prevent blackouts. Due to higher cost of installation, PMU cannot install at every power system buses. In this paper, Gravitational Search Methodology is introduces to optimize the placement of PMUs and provide the higher observability of power system buses. This new Gravitational Search Algorithm (GSA) based method has been applied to the IEEE 14-bus, IEEE 30-bus, IEEE 118-bus and Indian Northern Regional Power Grid (NRP) 246-bus test systems for normal and contingencies conditions both. The effectiveness of proposed methodology reveals optimal number of PMUs with redundant observability by proposed method.

**Keywords:** Phasor Measurement Unit, Law of Gravity, Gravitational Search Algorithm, Observability (Obs.), Optimal Placement.

## Introduction

Today, synchrophasor technology is the first priority in power industries because of their reliable supports and prevents blackouts. Wide Area Measurement System (WAMS) has grown from a handful of PMUs in the power grid. Introduction of phasor measurement units have facilitated producing time synchronized measurements of voltage and current signals facilitating real time synchronized measurements [1]. Day by day, the time frame of synchronized information has been persistently reduced from minutes to microseconds. In 1980s, the first prototypes of PMUs were developed at Virginia Tech and first commercial manufacture of PMUs with Virginia Tech collaboration was started by Macrodyne in 1991. PMU data are time-stamped with high precision at the source and used for wide area measurement systems (WAMS) applications by power engineers and system operators as a time synchronized tool. PMUs are highly accurate and advanced time synchronized technology which provides the voltage and current phasor and frequency information with Global Positioning System (GPS) receivers that allow the synchronization of the several readings taken at distance points. The PMUs at different buses collect voltage and current phasors and send that data to the Phasor Data Concentrator (PDC) and PDC send that to the super PDC where many other PDCs and directly PMUs are connected. Then super PDC send that data to the control center for advance applications of the power system. Selection of power system buses to install the PMU and get desired measurements is a challenging job. To see the dynamic view of bulk power grid, PMU is the most important and reliable real time measuring device.

There are many algorithms and approaches have been published in the literature for optimal placement of PMUs in power system. Initial work on PMU development and utilization has been reported by Phadke et al. [1-2]. In [3, 4], author proposed an algorithm which finds the minimal set of PMU placement needed for power system where the graph theory and simulated annealing method have been used. In [5], a strategic PMU placement algorithm is developed to improve the bad data processing capability of state estimation. Providing selected buses with PMUs can make the entire system observable. This will only be possible by proper placement of PMUs among the system buses. The authors in [6-10] developed an optimal placement algorithm for PMUs by using integer linear programming. In [12-14], a genetic algorithm is used to find out the optimal locations of PMUs. Reference [14], is a combination of immunity algorithm and genetic algorithm. In [15], a Tabu Search (TS) method is proposed for the OPP problem in which augmented incidence matrix is used for the observability analysis of PMU. A recursive TS method is suggested in [16] which is more superior than multiple Tabu Search.

The final results of OPP are in binary form hence researchers directly utilize the binary objective function. In [17, 18], a binary search method is proposed to find the optimal PMU placement. In [19], authors presented a binary particle swarm optimization (PSO) methodology for optimal placement of PMUs when using a mixed measurement set. A modified binary PSO algorithm is suggested in [20] for optimal placement of PMUs for state estimation. In [21], author used an iterated local search method to minimize the size of the PMU configuration needed to observe the power system network. In [22], author proposed the multi-objective problem of PMU placement and used non-dominated sorting differential evolution algorithm

based on pareto non-dominated sorting. An unconstrained nonlinear weighted least square algorithm is developed in [23] for optimal placement of PMUs.

This paper has been proposed Gravitational Search Algorithm (GSA) to minimize the number of PMUs with redundant observability (Obs.) for normal and contingencies both the conditions. The GSA is based on the Newton's Law of Gravity and mass interactions [24]. This methodology has been found high quality performance in solving many optimization problems in the literatures [11, 25-26]. In GSA, agents are considered as objects and their performance is measured by their masses. In GSA, heavy masses correspond to good solutions and move more slowly and conversely light masses correspond to weak solutions and fast move toward heavy masses. Finally heavy masses correspond to best solution. The objective of this paper is optimal placement of PMUs in order to achieve redundant observability.

The paper contains following discussion: First discussed the problem of minimum PMU placement in the power system then explain brief discussion of the GSA and presented the optimal PMU placement (OPP) by using GSA. Finally test results are given in this paper concludes the paper.

### PMU Formulation

This section formulates the objective of optimal placement of PMUs with maximum observability. Objective function in this paper contains two objectives, first is to find the minimum number of PMUs for full observability and second is to find the best location for maximum observability. Reference [11], has been used single objective function with the same Gravitational Search Methodology but probability to get maximum observability at same number of PMUs in next trial is comparatively low due to absence of maximum observability term in the objective function. This problem overcomes in this paper which is formulated as follows [7]:

$$\text{Min} \left[ \sum_{i=1}^N w_i z_i + \delta \sum_{i=1}^N (u_i - f_i) \right] \quad (1)$$

$$\text{Subject to, } f = AZ \geq b \quad (2)$$

$$Z = [z_1 z_2 \dots \dots \dots z_N]^T$$

where,  $N$  is total number of system buses,  $w_i$  is weight factor accounting to the cost of installed PMU at bus  $i$ ,  $A$  is a binary connectivity matrix of the system,  $Z$  is a binary variable vector having elements  $z_i$  define possibility of PMUs on a bus  $i$  i.e.  $x_i=1$ , if a PMU is needed at bus  $i$ , otherwise 0.  $AZ$  is a vector such that its entries are non-zero if the corresponding bus voltage is observable using the given measurement set and according to observability rules mentioned above. It ensures full network observability while minimizing the total installation cost of the PMUs, otherwise its entries are zero.  $b$  is a vector whose entries are all ones, it means each bus should be observed at least one time. If the entries in vector  $b$  is modified from one to two it means each bus should be observed at least two times then it provide the optimal location of PMUs in the case of single PMU outage or a single branch outage.  $\delta$  is normalizing coefficient for the observability redundancy. A suitable value of  $\delta$  should be chosen as equ. (3) because higher the value of  $\delta$  increases the number of PMUs.

$$\delta = \frac{1}{\sum_{i=1}^N u_i} \quad (3)$$

$u_i$  is the summation of  $i^{\text{th}}$  row in connectivity matrix ( $A$ ).  $f$  is the observability times of bus  $i$ . The entries in  $A$  are defined as follows:

$$a_{ij} = \begin{cases} 1 & \text{if } i = j \\ 1 & \text{if } i \text{ and } j \text{ are connected} \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

After getting the optimal number of PMUs, we can easily check the observability of each bus of the system and expression for total observability ( $O_{total}$ ) is given as:

$$O_{total} = \sum_{k=1}^P A_{L(k)} \quad (5)$$

where,  $P$  is the total optimal number of PMUs,  $A$  is the connectivity matrix, and  $L$  is the location of PMUs at the power system buses.

### Gravitational Search Algorithm (GSA)

Gravitational search algorithm is a heuristic algorithm that uses the concept of gravity and laws of motion to provide a suitable solution for an optimization problem. It is a well known fact that any two particles in the universe attract each other by a gravitational force directly proportional to the product of their masses and inversely proportional to the distance between them [24]. This concept is utilized in GSA along with the laws of motion, where agents are treated as objects and performance of agents is considered by their masses with gravitational force acting as a mode of communication between them.

The basic idea of GSA has been discussed in [11]. Now, consider a system with  $Q$  agents or masses. We define the position of the  $i_{th}$  agent by:

$$X_i = (x_i^1, \dots, x_i^d, \dots, x_i^n) \text{ for } i=1,2,3,\dots,Q \quad (6)$$

where,  $x_i^d$  represents the position of  $i_{th}$  agent in the  $d_{th}$  dimension. At a specific time 't', the force acting on mass 'i' from mass 'j' can be defined as following:

$$F_{ij}^d = G(t) \frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t) + \varepsilon} (x_j^d(t) - x_i^d(t)) \quad (7)$$

where  $M_{aj}$  is the active gravitational mass of agent  $j$ ,  $M_{pi}$  the passive gravitational mass of agent  $i$ ,  $G(t)$  gravitational constant at time  $t$ ,  $\varepsilon$  a small constant, and  $R_{ij}(t)$  is the Euclidean distance between two agents  $i$  and  $j$ . The total force acting on the  $i_{th}$  agent ( $F_i^d(t)$ ) is calculated as follows:

$$F_i^d(t) = \sum_{j \in K_{best}, j \neq i} rand_j F_{ij}^d(t) \quad (8)$$

where  $rand_j$  is a random number in the interval  $[0, 1]$  and  $k_{best}$  is the set of first  $K$  agents with the best fitness value and biggest mass.  $k_{best}$  is a function of time, with the initial value of  $K_0$  at the beginning and decreasing with time. In such a way, at the beginning, all agents apply the force, and as time passes,  $k_{best}$  is decreased linearly and at the end there will be just one agent applying force to the others.

By the law of motion, the acceleration of agent  $i$  at time  $t$ , in direction  $d_{th}$  is given by:

$$Ac_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)} \quad (9)$$

where  $M_{ii}$  is the inertial mass of  $i_{th}$  agent.

Furthermore, the next velocity of an agent is a function of its current velocity added to its current acceleration. Therefore, the next position and the next velocity of an agent can be calculated as follows:

$$v_i^d(t+1) = rand_i \times v_i^d(t) + Ac_i^d(t) \quad (10)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \quad (11)$$

where  $rand_i$  is a uniform random variable in the interval  $[0, 1]$ . The gravitational constant  $G$ , is initialized at the beginning of problem and will be decreased with time to control the search accuracy [24].

$$G(t) = G_0 e^{-\alpha \frac{t}{T}} \quad (12)$$

where  $\alpha$  is a user specified constant,  $t$  is the current iteration and  $T$  is the total iterations. Gravitational and inertial masses are simply calculated by the fitness evaluation. Both of the masses namely gravitational and inertial are assumed to be equal and their values are calculated using the map of fitness. The gravitational and inertial masses are updated by using following equations:

$$M_{ai} = M_{pi} = M_{ii} = M_i \quad (13)$$

where,  $i=1, 2, 3, \dots, Q$

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \quad (14)$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^Q m_j(t)} \quad (15)$$

where  $fit_i(t)$  represents the fitness value of the agent  $i$  at time  $t$ , and the  $best(t)$  and  $worst(t)$  in the population of agents respectively indicate the strongest and the weakest agent according to their fitness and can be defined as follows:  
For a minimization problem:

$$best(t) = \min_{j \in \{1, \dots, Q\}} fit_j(t) \quad (16)$$

$$worst(t) = \max_{j \in \{1, \dots, Q\}} fit_j(t) \quad (17)$$

The procedure to apply the GSA for solving the OPP problem is as follow:

- Step 1. Read bus data and line data of the test system.
- Step 2. Find the connectivity matrix ( $A$ ) from line data.
- Step 3. Initialize GSA parameters  $T$ ,  $Q$ ,  $G_o$  and  $\alpha$ .
- Step 4. Identify the search space.
- Step 5. Generate initial population between minimum and maximum values of the control variables.
- Step 6. The fitness values of each agent in the population are calculated for the OPP problem.
- Step 7. Based on fitness value, update  $G(t)$ ,  $best(t)$ ,  $worst(t)$  and  $M_i(t)$  for  $i=1, 2, \dots, Q$
- Step 8. Calculation of total force in different directions using Eq. 8.
- Step 9. Acceleration of each agent is modified using Eq. 9.
- Step 10. The velocity and position of each agent is updated using Eq. 10 and Eq. 11 respectively.
- Step 11. Repeat steps 6-10 until the stop criterion is reached.
- Step 12. Stop

## Case Study

This paper, proposed a simple case of power system which has been applied to check the effectiveness of proposed methodology for both the simple and contingency conditions. Single PMU outage and single line outage have been considered as a contingency in this paper. Finally the objective function of proposed method has been compiled in Newton method based GSA which is the new heuristic optimization technique. Fig. 1 and Fig. 2 show the single line diagrams of IEEE 14-bus and IEEE 30-bus test systems obtained from DIGSILENT software respectively. All the experiments are executed on a computer having the following configuration: Intel core i3 CPU @ 3.40 GHz, 2 GB RAM.

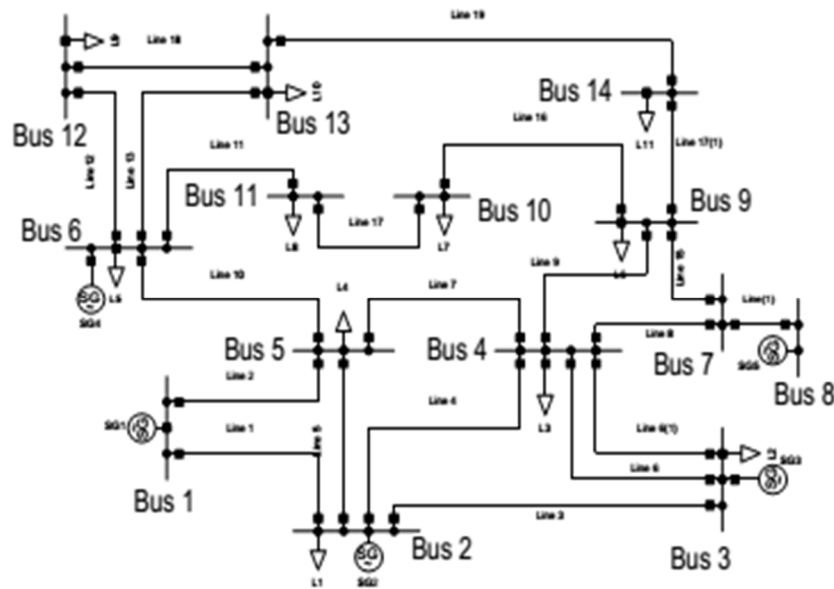


Figure 1. IEEE 14-bus test system

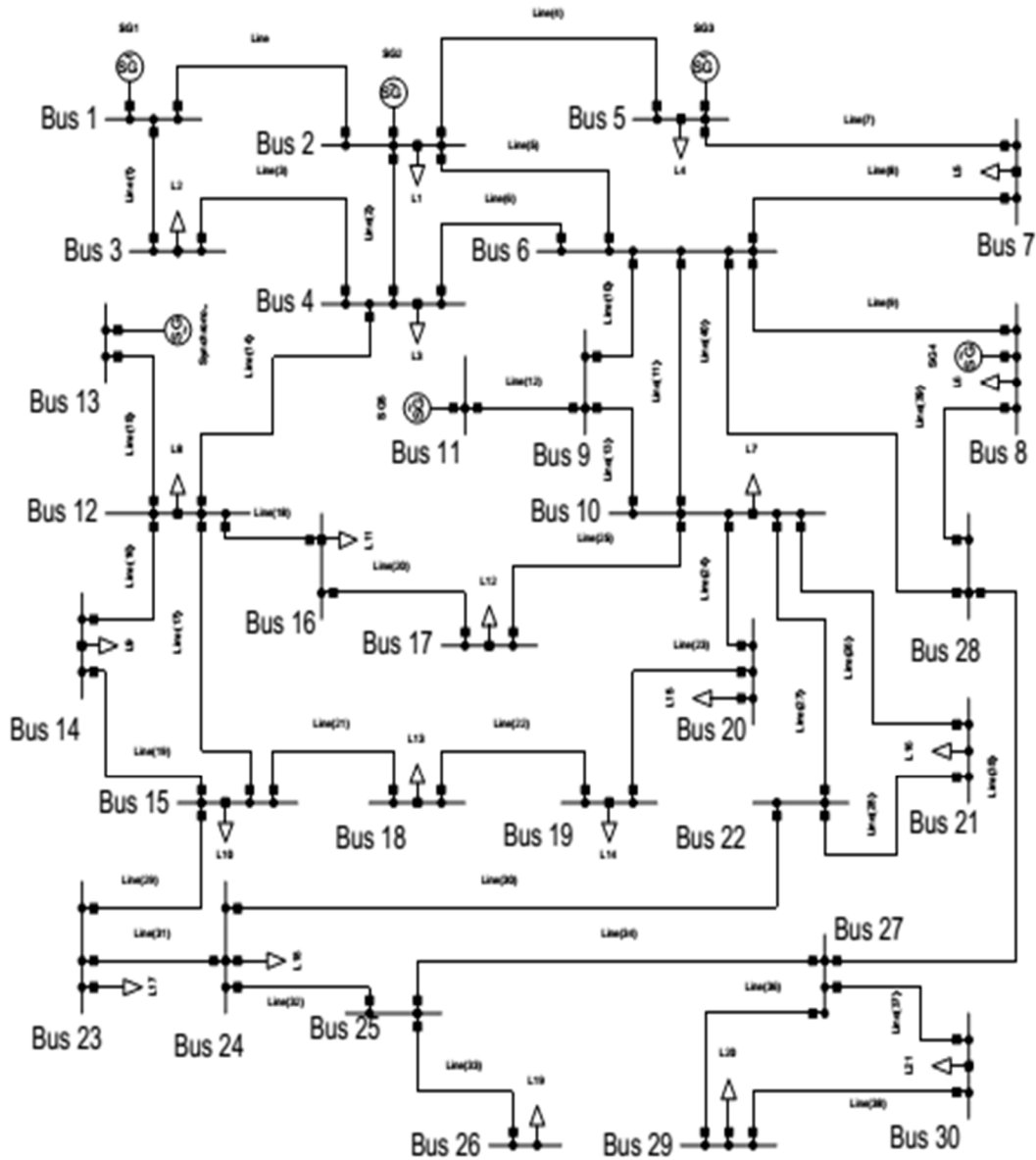


Figure 2. IEEE 30-bus test system

Table 1 shows the chosen values of the parameters for the GSA used for the OPP problem solution. To find out the best result, selections of GSA parameters play an important role. These values have been arrived at by various trials of OPP solutions on all the system tested. The values listed in Table 1 produced best performance in terms of finding the optimal solution and computational time.

Table 2 shows the optimal number of PMUs and their locations which provide the full observability of power network. Fig. 3 shows the comparison of convergence curve of GSA for the IEEE 118-bus system. A steep decline in objective function value is observed in Fig. 3. The computation times of proposed OPP method for IEEE 14-bus, IEEE 30-bus, IEEE 118-bus and NRPG 246-bus [27] test systems are 0.60 sec, 0.76 sec, 3.32 sec and 15.3 sec respectively. It is observed from Fig. 3, the GSA converged in seventy three iterations for ref. [11] and suggested thirty two PMUs for full observability of the power system. But same problem with proposed methodology suggested same number of PMUs in less convergence of GSA as compared to [11] and probability to get same results (maximum Obs.) is fixed with proposed methodology which is the main advantage over the ref. [11].

The results of proposed algorithm have been compared with the previously reported methods in the literature and shown in Table 3. For all the test systems, minimum numbers of PMUs are same as the previously reported methods but the locations of PMUs by proposed method are different which provide the redundant observability of each bus as shown in Table 4.

Table 1. GSA Parameter

$G_0$	$\alpha$	$Iter_{max}$	$Q$
1	20	150	50

Table 2. Optimal Location of PMUs for Test Systems

System configuration	Minimum no. of PMUs	Optimal PMU Locations	CPU Time (s)
IEEE 14-bus	4	2, 6, 7, 9	0.60
IEEE 30-bus	10	2, 4, 6, 9, 10, 12, 15, 20, 25, 27	0.76
IEEE 118-bus	32	3, 5, 9, 11, 12, 17, 21, 25, 28, 34, 37, 41, 45, 49, 52, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114	6.32
NRPG 246-bus	70	6,21,23,24,29,34,40,45,48,54,55,57,60,61,62,63,65,69,73,74,75,78,80, 88,93,95,98,100,101,102,103,106,109,116,117,121,122,125,126,129,132,134,140,141,142,147,157,158,160,163,168,173,181,183,185,187,190,191,194,199,201,202,203,215,216,219,234,235,243,245	15.3

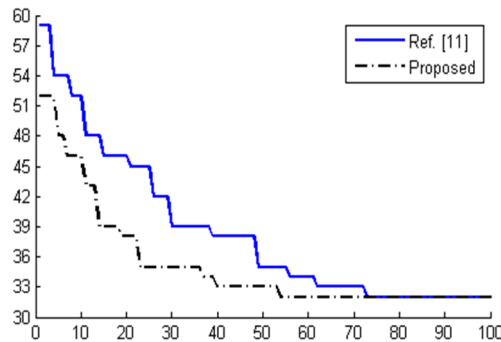


Figure 3. Convergence of GSA for IEEE 118-bus test system

Table 3. Comparison of Obtained Results by Several Methods

Test System	IEEE 14-Bus	IEEE 30-Bus	IEEE 118-Bus	NRPG 246-Bus
Proposed method	4	10	32	70
Generalized ILP [8]	4	10	NA	NA
Ref. [11]	4	10	32	NA
Binary Search Algorithm [17]	4	10	--	NA
WLS [23]	4	10	32	NA

Table 4. Comparison of obtained results on the basis of each bus observability for IEEE 30-bus

	Observability (Obs.) of each bus	Total observability
Proposed method	1 3 1 4 1 5 1 1 3 4 1 3 1 2 2 1 1 1 1 2 1 1 1 1 2 1 2 2 1 1	52
Ref. [11]	1 3 1 4 1 5 1 1 3 3 1 3 1 2 2 1 1 2 1 2 1 1 1 1 2 1 2 2 1 1	52
Ref. [17]	2 3 1 3 1 4 1 1 3 3 1 2 1 2 2 1 1 2 1 2 1 1 1 1 2 1 2 2 1 1	50
Ref. [23]	1 1 1 1 1 3 1 1 2 1 1 1 1 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	35

After the modification of entries in vector  $b$  of Equ. (2), proposed method provides the optimal PMU placement in the case of single PMU outage. If the system is observable in case of single PMU outage it means system will also be observable in case of single branch outage, because single branch outage case is the subset of single PMU outage case. Table 5 shows the results in case of single PMU outage or single branch outage with comparison to other method [6]. For all the IEEE test systems, and Indian NRPG 246-bus system, proposed method provides the maximum observability as compared to [6] in Table 5. Also the

Table 5. OPP in case of single PMU outage or single branch outage case with comparison

Test System	IEEE 14-Bus		IEEE 30-Bus		IEEE 118-Bus		NRPG 246-bus	
	No. of PMUs	Total Obs.	No. of PMUs	Total Obs.	No. of PMUs	Total Obs.	No. of PMUs	Total Obs.
<b>Proposed method</b>	9	39	21	85	68	305	153	614
<b>Ref. [6]</b>	9	34	22	85	72	316	NA	NA

number of PMUs in IEEE 30-bus and 118-bus systems are minimum with maximum observability. In this case, computation time for IEEE 14-bus and IEEE 30-bus is around same as previous case but for IEEE 118-bus system and for Indian NRPG 246-bus system it is 7.8 sec and 17.9 sec respectively.

## Conclusion

This paper presented a Gravitational Search method for solving the multiobjective OPP problem, which minimizes the total number of PMUs and provides the maximum observability of the power system for both the normal and contingencies conditions. The method was applied on IEEE 14-bus, IEEE 30-bus, IEEE 118-bus and Indian Northern Regional Power Grid (NRPG) 246-bus test systems and its results are compared with other methods reported in the literatures. The simulation results and fast convergence time indicate that the proposed algorithm satisfactorily provides maximum observable system measurements with minimum number of PMUs. The maximum observability of proposed methodology was found to be better than other methods.

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